Algorithms for single or multi-level optimization with black-box terms

Optimization of decentralized systems, where multiple decision-makers are involved, can be seen as a game where players pursue their own interests while operating within a shared environment. In some cases, when players are organized hierarchically or decisions are made sequentially, earlier decisions made by one player become visible to later players, so-called Stackelberg games. These games can be modeled as multi-level optimization problems, where each level represents a specific player, and the decisions made by higher-level players are considered as input parameters for lower-level decisions. From the complexity theory point of view, this class of optimization problems is harder than the well-known NP-complete problems [1-3]. The theory around such kind of optimization models steadily expands, and one may gather a detailed picture of state-of-the-art from recent reviews [4,5]. Despite a tremendous progress in solution techniques, the demands of practical applications significantly surpass the algorithms' capabilities. Further progress is highly desired here. The present challenge aims to highlight a promising and demanded direction of research in this area.

Advanced applications often consider systems that involve components that cannot be easily modeled using conventional algebraic expressions suitable for in-depth analysis or general-purpose solvers. These components of the system can be represented by a black-box function that must be invoked to evaluate the viability or quality of a candidate solution. From a practical point of view, a model incorporating black-box components as part of its design is significantly more flexible, as these black-box components can be substituted for alternative ones, enabling a wide range of potential applications for the model, while preserving its overall structure and, consequently, a set of tools dealing with it.

A literature on multi-level optimization models with black-box terms is scarce due to the fact that this research topic is on its very beginning. One may highlight a work [6], where bi-level models with black-box constraints on the lower-level is studied. The authors consider gas market models with chance constraints as an application for their method. Great practical potential along with almost countless number of yet unresolved difficulties associated with the models of this class substantiate the importance and complexity of the present challenge. In its scope, the list of promising directions of work could include but is not limited to:

1. Literature review on multi-level optimization models with black-box terms in objective functions of the decision makers or/and the constraints in each or at least one level of the hierarchical system. All aspects related to algorithms and complexity are interesting and important including single-level optimization models with black-box terms, for example, with simulation optimization models.
2. Development of optimization algorithms for the single or multi-level optimization models with black-box terms. Exact methods, approximate algorithms and metaheuristics, including hybrids with classical mathematical optimization methods, are highly desirable. A posteriori upper and lower bounds for the optimal solution, if they exist, can be studied for the large-scale models.
3. Parallel versions of optimization algorithms on CPUs, GPUs, and NPUs are special interests of the project. Distributed methods, multi-agent systems, decomposition methods, and other well-established approaches can be adapted for some special cases of the single or multi-level optimization models with black-box terms in constraints or objective functions.

4. Feasibility and optimality concepts for the case of multiple optimal solutions for the lower-level problems should be defined and studied. Optimistic and pessimistic feasibility concepts must be incorporated into the algorithmic frameworks for the models with black-box terms. The existence of an optimal solution for the mixed integer models remains an important open question.
5. As a general rule, each decision-maker has multiple objectives, and in practice, we may face extremely difficult cases, where multiple followers make their own decisions at lower levels. The concepts of feasibility and optimality should be modified for these cases and studied from algorithmic perspective.

We would greatly appreciate any insights, recommendations, or innovative approaches that could assist in addressing this challenge. Your expertise and contributions could significantly enhance our collaborative efforts, paving the way for deeper exploration and fruitful future partnerships.

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